QUINE'S DOUBLE STANDARD: UNDERMINING THE INDISPENSABILITY ARGUMENT VIA THE INDETERMINACY OF REFERENCE

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Abstract

Quine has famously put forward the indispensability argument to force belief in the existence of mathematical objects (such as classes) due to their indispensability to our best theories of the world (Quine 1960). Quine has also advocated the indeterminacy of reference argument, according to which reference is dramatically indeterminate: given a language, there's no unique reference relation for that language (see Quine 1969a). In this paper, I argue that these two arguments are in conflict with each other. Whereas the indispensability argument supports realism about mathematics, the indeterminacy of reference argument, when applied to mathematics, provides a powerful strategy in support of mathematical anti-realism. I conclude the paper by indicating why the indeterminacy of reference phenomenon should be preferred over the considerations regarding indispensability. In the end, even the Quinean shouldn't be a realist (platonist) about mathematics.

1. Introduction

Among the various arguments that W. V. Quine developed, two deserve particular attention: the indispensability argument (see, e.g., Quine 1960, and Putnam 1971) and the argument for the indeterminacy of reference (see Quine 1969*a*). The indispensability argument establishes that we ought to be committed to the existence of mathematical entities, given that they are indispensable to our best scientific theories.[1] The argument for the indeterminacy of reference establishes that reference is substantially indeterminate. After all, given any reference relation for a language, it's always possible to generate an alterna-

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tive reference relation that fits the data in question just as well as the original relation.[2]

In this paper, I argue for two main claims. First, there is a tension between these two arguments. Whereas the indispensability argument is an argument for *realism* about mathematics, the indeterminacy of reference argument—when applied to mathematical notions—is a powerful *anti*-realist argument. Second, if we had to choose between these two arguments, I argue that priority should be given to the indeterminacy of reference argument. This argument underscores a much more general phenomenon than the indispensability argument—a phenomenon that applies to the whole of our language, including the language used to state our best theories of the world. As a result, the indeterminacy ends up *undermining* the indispensability argument, thus providing a powerful *response* to those who claim that mathematics is indispensable. As a result, the Quinean should give up the indispensability argument and embrace anti-realism about mathematics (a position that Quine himself initially entertained; see Goodman and Quine 1947).

2. The Indispensability Argument

According to the indispensability argument, given that mathematical objects are indispensable to our best theories of the world, we ought to be ontologically committed to their existence. Mark Colyvan presented the argument in a very clear and interesting way (see Colyvan 2001):

> (P1) We ought to be ontologically committed to all and only the entities that are indispensable to our best scientific theories.
> (P2) Mathematical entities are indispensable to our best scientific theories.
>
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> Therefore, we ought to be ontologically committed to mathematical entities.

Formulated in this way, Colyvan notes, the argument has two crucial assumptions (Coyvan 2001). One is *confirmational holism*, according to which scientific theories are confirmed or disconfirmed not as independent, separate hypotheses, but as wholes (Quine 1961*a*). In this way, the empirical data that confirm a theory confirm it as a whole, including any mathematics that is used in the formulation of the theory. Hence, *all* entities that are indispensable to our best scientific theories should be taken to exist. The second assumption is *naturalism*, according to which to determine what exists, we should refer to our best scientific theories. In fact, for the naturalist, *only* the entities that are indispensable to our best theories of the world should be taken to exist. Scientific methodology is taken to provide a reliable strategy to generate good theories about the world, and what is not positively countenanced by science should not be taken to exist. In this way, all and only the entities that are indispensable to our best theories of the world should be taken to exist.

Naturalism and, in particular, confirmational holism are often tied to an additional component: the *underdetermination argument*. Since we can't separate and evaluate isolated hypotheses, we have to consider them as "whole blocks". And if we are willing to make changes elsewhere in the system, we can always reconcile different (incompatible) theories about unobservable objects with the data. Hence, the underdetermination argument goes, the data don't uniquely determine a theory—radically different theories are often compatible with the available evidence, and often there's no purely empirical way of deciding between such theories (see Quine 1961*a*).

However, this immediately raises a puzzle. The underdetermination argument is a common feature of various empiricist views. For example, both Quine and van Fraassen have insisted on the fact that typically the data don't uniquely determine a theory (see, e.g., Quine 1961*a*, and van Fraassen 1980). Moreover, according to van Fraassen, "to be an empiricist is to withhold belief in anything that goes beyond the actual, *observable* phenomena" (1980, pp. 202–203; italics added). But if the indispensability argument is used, the empiricist ends up believing in the existence of abstract entities, such as sets, functions and numbers. Since the latter are *un*observable (*pace* Maddy 1990), the empiricist's set of beliefs turns out to be incoherent. How can the empiricist overcome this?

Well, it might be argued that this is *not* a problem for Quine. He *wouldn't* accept van Fraassen's constraint that to be an empiricist is to "withhold belief" in *un*observable entities. One of the main *outcomes* of the indispensability argument is to make room, and to provide a reason, for empiricists to believe in unobservable objects—whether they are abstract, such as sets and numbers, or concrete, such as photons and electrons.[3] As long as the objects are indispensable to our best scientific theories, insists Quine, such theories are ontologically committed to them.[4]

Quine's commitment to the indispensability argument is uncontroversial. As he points out:

> Ordinary interpreted scientific discourse is as irredeemably committed to abstract objects—to nations, species, numbers, functions, sets—as it is to apples and other bodies. All these things figure as *values of the variables* in our *overall system of the world*. [See the first premise of the indispensability argument, (P1), above.] The numbers and functions contribute *just as genuinely* to physical theory as do hypothetical particles. [See (P2), above.] (Quine 1981, pp. 149–150; italics added.)

Putnam presented the same point, just more explicitly:

> Quantification over mathematical entities is *indispensable* for science, both formal and physical [see (P2), above]; therefore we should *accept such quantification* [see (P1), above]; but this *commits us to accepting the existence of the mathematical entities* in question. This type of argument stems, of course, from Quine, who has for years stressed both the *indispensability* of quantification over mathematical entities and the *intellectual dishonesty* of denying the existence of what one daily presupposes. (Putnam 1971, p. 347; italics added.)

As a result, given the indispensability argument, Quine (rather grudgingly) accepts the introduction of mathematical objects into his ontology. This becomes particularly clear in *Word and Object*, where Quine argues for the need for classes (sets) in science (Quine 1960). But, he insists, *only* this type of abstract object—that is, classes—is needed, given that all mathematical objects that are required in science can be characterized in terms of them. In other words, given Quine's predilection for desert landscapes, when he posits abstract entities, he tries to minimize the *types* of objects that are introduced (that is, only classes are accepted, nothing else). However, does Quine really have good grounds to posit classes?

3. The Indeterminacy of Reference Argument

To examine this question, we need to consider Quine's answers to two related questions: (a) How can we *uniquely* determine the objects our terms *refer* to? (b) How can we determine, in *absolute* terms, what objects *there are*? In each case, Quine's answer is clear: we simply can't! But, as will become clear shortly, with this answer, even the postulation of classes is jeopardized.[5] After all, if there is no absolute answer to the question regarding what objects exist, there is, in particular, no absolute answer regarding the existence of classes. So, is Quine really justified in positing such objects? Before examining this issue (which is the subject of next section), it's crucial, first, to scrutinize the arguments Quine employs to answer (a) and (b).

Quine's response to (a) is given by the indeterminacy of reference argument; his answer to (b) emerges from the related argument for ontological relativity. The argument for the indeterminacy of reference establishes that reference is radically indeterminate, and so there is no guarantee that we can uniquely single out the referents of our terms. The ontological relativity argument, in turn, establishes that there is no absolute answer to the question of what exists. After all, any answer to this question is ultimately relative to the individuation criteria and associated features of the conceptual framework we use. The outcome of these arguments is plain: ontological questions and questions about reference are radically indeterminate.

It might be argued that it's not obvious whether the argument for the indeterminacy of reference is actually an *argument*, rather than an *idealized description* of an aspect of our language use. In this sense, Quine's considerations about referential indeterminacy can't be conclusive. At best, they sketch a phenomenon that may or may not be pervasive, but no conclusive evidence for this fact is actually provided.

In response, it seems that Quine does put forward an *argument* for the indeterminacy of reference. In fact, Quine's argument is an exemplar of a particular style of philosophical reasoning in which idealized considerations are introduced to identify general features of a certain phenomenon. The focus on idealized features of the phenomenon gives the impression that instead of an argument Quine only has a simple idealized description of the phenomenon. But this way of conceiving of the argument misunderstands Quine's strategy. It is crucial for Quine that the state of affairs he considers be *plausible*, *possible*, and *indistinguishable* from the scenario postulated by those who believe that reference is determined. In fact, the indeterminacy argument is a skeptical argument. It's sufficient for Quine's purposes to introduce a plausible scenario, empirically equivalent to the scenario in which reference is determined, but in which reference *is actually indeterminate*. This shows that nothing in our practice rules out the radical indeterminacy of reference. Quine's indeterminacy argument is, ultimately, an *underdetermination* argument.

In this sense, the argument for the indeterminacy of reference has much in common with several other celebrated philosophical arguments. For example, Hobbes' argument for the emergence of the State based on a fundamental state of nature provides an *idealized*, *plausible description* of how the State could have emerged to make a philosophical point about the State's legitimacy. Descartes' evil demon argument or Nozick's brains in a vat scenario provide *underdetermination* arguments to challenge the possibility of our knowledge of the external world. In each of these cases, a response that insists that we need to know that the scenarios considered by each argument are actually true misses the point.

Quine's scenario is now familiar. Imagine we are visiting a foreign land, and we don't know the language of the native people. We are trying to learn that language, and we consider what is the native's term for rabbit. But, notes Quine, we immediately face a predicament:

> A whole rabbit is present when and only when an *undetached part* of a

> rabbit is present; also when and only when a *temporal stage* of a rabbit is present. If we are wondering whether to translate a native expression 'gavagai' as 'rabbit' or as 'undetached rabbit part' or as 'rabbit stage', we can never settle the matter simply by ostension—that is, simply by repeatedly querying the expression 'gavagai' for the native's assent or dissent in the presence of assorted stimulations. (Quine 1969*a*, pp. 46–47.)

Given that a whole rabbit, an undetached part of a rabbit and a temporal stage of a rabbit all have the same extension, we cannot decide, based on the native's behavior alone, whether 'gavagai' is correctly translated by 'rabbit', 'undetached rabbit part' or 'rabbit stage'. The native's behavior—e.g. his or her assent to the expression 'gavagai' in various circumstances—is insufficient to single out the meaning of the foreign expression, given the compatibility of the native's behavior with at least three different uses, three different meanings of the term. This fact highlights the underdetermination that underlies the indeterminacy argument.

But as Quine is eager to point out, the indeterminacy here is not only of meaning, but of reference as well. Clearly, 'rabbit', 'undetached rabbit part' and 'rabbit stage' have different meanings, and if we are unable to determine which of these terms (if any) provides the proper translation of 'gavagai', we are unable to determine to which object 'gavagai' *refers* after all. Indeed, typically to be able to determine the reference of a term seems to be a necessary condition for the determination of its meaning—even though it's clearly not sufficient. As Quine insists:

> It is philosophically interesting [...] that what is *indeterminate* in this artificial example is not just meaning, but *extension*; *reference*. [...] The terms 'rabbit', 'undetached rabbit part', and 'rabbit stage' differ not only in meaning; *they are true of different things*. *Reference itself proves behaviorally inscrutable*. (Quine 1969*a*, p. 48; italics added.)

Given the impossibility of behaviorally determining which scenario is the case—that is, which term correctly translates 'gavagai', what exactly its meaning is—reference does become inscrutable.

Someone may complain that the situation is not as dramatic as the example suggests. After all, *relative to the apparatus of individuation of a background theory*, it is possible to distinguish between 'rabbit', 'undetached rabbit part', and 'rabbit stage'. For example, if we share with the natives the same notion of identity, and if we are able to communicate with them using that notion, we can, e.g., ask the natives whether this 'gavagai' is *the same* as that one. In this way, by appropriately querying the natives using the individuation criteria of a background theory, we could eventually uniquely determine the proper referent of that term.

Quine doesn't fail to note the trouble with this suggestion, though. The problem is that *the individuation apparatus itself might be indeterminate*. In this case, there's no hope to uniquely determine the referents of the terms in the language in question, given that different individuation apparatuses yield different answers regarding what there is. For example, we can't pretend to have uniquely answered the question of what exists by simply making a list: there are oceans, sunsets, clouds, holes, rabbits, shadows, left fingers, magnolias… After all, the list itself *presupposes a way of individuating each object*, and depending on the individuation criteria one uses, a *different* list is produced.

Moreover, the complaint above assumes that we could share with the natives the same notion of identity. But to be able to share the *same* identity notion, not only would we need to have a common notion of identity, but also have some way of comparing identity notions and determining whether they are *the same*. But this *presupposes*, of course, a notion of identity—and a regress starts. To which of these identity notions are we referring when it's considered that we share *the same* identity notion with the natives? There's no way of telling without assuming that the individuation apparatus is *not indeterminate*, in that the apparatus has a unique notion of identity across the board. But whether this is the case—that is, whether ultimately the individuation apparatus is or is not indeterminate—is precisely the point in question.

It might be argued that Quine's 'gavagai' scenario only emerges because we are dealing with certain objects in the physical world that are notoriously elusive. For example, is a rabbit minus one of its hairs still *the same* rabbit? Where do a cloud end and a new one start? But nothing of this sort will ever emerge if we consider stable, precise objects, such mathematical entities.

Quine disagrees, of course. On his view, the indeterminacy extends even to mathematics. In fact, referential indeterminacy is already present in one of the most basic parts of mathematics: arithmetic. After all, there's no unique way of characterizing the natural numbers. As Quine points out (following Benacerraf 1965):

> Consider now the arithmetician himself, with his elementary number theory. His universe comprises the natural numbers outright. [...] [But] what, after all, is a natural number? There are Frege's version, Zermelo's, and von Neumann's, and countless further alternatives, all *mutually incompatible* and *equally correct*. (Quine 1969*a*, p. 51; italics added.)

Quine is here referring to the familiar reconstructions of arithmetic in set theory (or, in Frege's case, in second-order logic). In Zermelo's case, e.g., we have the following characterization:

$0 = \varnothing$
$1 = \{\varnothing\}$
$2 = \{\{\varnothing\}\}$
$3 = \{\{\{\varnothing\}\}\}$
$\dots$

In von Neumann's case, in turn, we have:

$0 = \varnothing$
$1 = \{\varnothing\}$
$2 = \{\varnothing, \{\varnothing\}\}$
$3 = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$
$\dots$

For Frege:

$0 = \#[x\colon x \neq x]$. (0 is the number of the concept *being non-self-*

identical.)

$1 = \#[x: x = 0]$. (1 is the number of the concept *being identical with the number* 0.)

$2 = \#[x: x = 0 \vee x = 1]$. (2 is the number of the concept *being identical with 0 or 1*.)

$3 = \#[x: x = 0 \vee x = 1 \vee x = 2]$. (3 is the number of the concept *being identical with 0 or 1 or 2*.)

…

When fully developed, any of these reconstructions yields the same results with regard to the properties of natural numbers.[6] In this sense, as Quine notes, they are all *equally correct*. But they are also *mutually incompatible*: the number 2 corresponds to radically different sets in Zermelo's and von Neumann's reconstructions, and it's not even a set in Frege's.

The outcome is that, even in mathematics, reference becomes indeterminate. When we refer to numbers, it's unclear what exactly we are referring to. As noted above, numbers can be identified with dramatically different sets or with objects that fall under certain concepts. Nothing in mathematical practice uniquely determines which objects numbers are. The style of argument here is clear. Once again, we have an underdetermination argument: the same "phenomena" (the various properties that numbers have) are compatible with radically different underlying objects (different reconstructions of arithmetic in set theory and in second-order logic).

But, perhaps, one could try to undercut the underdetermination by noting that it doesn't really matter how numbers are actually characterized. *Any* characterization will do. Numbers are whatever ends up satisfying arithmetic—whether these things are sets, Fregean objects, or what have you. The particular nature of numbers is actually irrelevant. What counts is the resulting *structure*, and this is something that all proposed reconstructions of arithmetic have in common. For example, when properly developed, such reconstructions all have a first element, a successor function and an appropriate comprehension principle. Not surprisingly, this view is called *structuralism*.

Quine, however, is skeptical of the idea that structuralism could undermine the referential indeterminacy of mathematics. As he is eager to point out:

> It is […] true to say, as mathematicians often do, that *arithmetic is all there is to number*. But it would be a *confusion* to express this point by saying, as is sometimes said, that *numbers are any things fulfilling arithmetic*. This formulation is *wrong* because *distinct domains of objects yield distinct models of arithmetic*. Any progression can be made to serve; and to identify all progressions with one another, e.g., to identify the progression of odd numbers with the progression of evens, would contradict arithmetic. (Quine 1969*a*, p. 52; italics added.)

In other words, even if we grant that arithmetic is all there is to number, there is still more to numbers than simply fulfilling arithmetic. After all, depending on the domain of objects one considers, *different* models of arithmetic emerge, and on pain of incoherence, such models *cannot* be identified. In fact, in the case of *first-order* arithmetic, the models in question need not be isomorphic, and so we cannot say that they are even *structurally the same*. Similarly, in the case of *second-order* arithmetic, there will be different models of the underlying second-order logic, depending on the semantics one considers: for example, standard or Henkin semantics. Although second-order arithmetic, when *restricted* to standard semantics alone, is categorical, the same is not the case if we use Henkin semantics. (The latter gives second-order logic the same metalogical properties of first-order logic, including the existence of nonstandard models.) As a result, if we consider *all the semantics available*, the models of second-order arithmetic will not be structurally the same either.

In other words, with the gavagai example and the case of arithmetic, it's unclear how one could uniquely determine the reference of our terms. On Quine's picture, however, referential indeterminacy emerges from a still more general phenomenon; namely, the fact that there's *no absolute answer* to the question about *what there is*. (This is, of course, Quine's ontological relativity doctrine.) Given that we can't answer ontological questions in absolute terms, it's not surprising that we can't uniquely determine which objects our terms refer to. As Quine insists:

> The relativistic thesis to which we have come is this […]: *it makes no sense to say that the objects of a theory are, beyond saying how to interpret or reinterpret that theory in another*. Suppose we are working within a theory and thus treating of its objects. We do so by using the variables of the theory, whose values those objects are, though *there be no ultimate sense in which that universe can have been specified*. In the language of the theory, there are predicates by which to distinguish portions of this universe from other portions, and this predicates differ from one another purely in the roles they play in the laws of the theory. Within this background theory we can show how some subordinate theory, whose universe is some portion of the background universe, can by a reinterpretation be reduced to another subordinate theory whose universe is some lesser portion. Such talk of subordinate theories and their ontologies *is* meaningful, *but only relative to the background theory* with its *own primitively adopted* and *ultimately unscrutable ontology*. (Quine 1969*a*, p. 54; italics added.)

In other words, ontological talk—talk about which objects there are—is always relative to a given background theory. There's no ultimate sense in which we could specify the objects that constitute the universe.

In conclusion, according to Quine, the indeterminacy of reference and ontological relativity go hand in hand. There is no absolute answer to the question regarding what there is, and no way of uniquely determining what our terms refer to. It's indeterminacy all the way down.

4. The Tension between the Two Arguments

Faced with the indispensability argument, on the one hand, and the arguments for the indeterminacy of reference and ontological relativity, on the other, a problem immediately arises. As we saw, according to the *indispensability* argument, we ought to be committed to the existence of mathematical entities—at least those that are indispensable to our best scientific theories. The outcome of the *indeterminacy of reference* and *ontological relativity* arguments, however, is that ontological issues cannot be settled in absolute terms. What there is, is *relative to a given background theory*, which includes, among other items, principles of identity and individuation for the objects in question. But this means that whether numbers, in particular, exist—and are Zermelo sets, von Neumann sets or Fregean objects—is also *relative* to a given background theory. If we shift the background theory, say, from von Neumann's to Frege's reconstruction, a very different answer regarding the *nature* of numbers will emerge. In this case, instead of conceiving of numbers as sets, we would take them to be objects that fall under certain concepts. If we move, in turn, to a modal-structural interpretation, then even the *existence* of numbers is no longer asserted. Only the *possibility* of certain structures is found. In other words, not even the commitment to numbers is forced on us by mathematics. But, as noted above, all of these dramatically different descriptions are *equally correct*, and they provide perfectly workable reconstructions of arithmetic. As a result, the dependence of ontological questions on a background theory seems to undermine the commitment to the *existence* of numbers that the indispensability argument brings, and the difficulty of uniquely determining the referents of our mathematical terms seem to undercut any proposed answer to the question regarding the *nature* of such numbers.[7]

In other words, the indispensability argument will make us believe in the *existence of mathematical entities*. In contrast, the indeterminacy of reference argument and the doctrine of ontological relativity highlight the fact that there is *no absolute way* of determining whether *there are* numbers and what their *nature* is. What one argument gives the others take back. Clearly, they are in tension with each other.

It might be objected that, *at best*, the indispensability argument establishes the *existence* of mathematical entities. It has never been part of the argument to try to settle the issue regarding the *nature* of such objects. In fact, the argument leaves it completely open what such objects are (see Colyvan 2001).

This is a fair point. But it comes at a price. What sort of realism about mathematics does the indispensability argument support? What exactly are the *mathematical objects* that the indispensability argument makes us believe in? Are they classes, sets, functions? As will become clear below, we *can't know*. The answer will depend on the *background theory* we use, and *radically different*, but *perfectly correct*, answers emerge with different background theories.

Note that if the indispensability argument establishes at best the *existence* of mathematical entities, without ever determining their nature, this is a significant *shortcoming* of the argument. If, at best, we can say that *there are* mathematical objects, but we cannot say anything about what kinds of objects they are—whether they are concrete or abstract, whether they are sets, functions, categories or something else altogether—it is simply *unclear* what the mathematical realist (or platonist) *is realist about*. And it isn't reasonable to be realist about objects whose nature, given the indeterminacy argument, we are unable to determine. The *content* of the realist claim, in this context, is radically indeterminate. And as will also become clear below, the tension between the indispensability and the indeterminacy arguments undermine any hope for any interesting form of realism about mathematics. In the end, it's an *ersatz* form of realism the one that insists that we should believe in the existence of objects whose nature we will never be able to know.

It might be argued that this is not the case. The sort of realism about mathematics that the indispensability argument supports is actually strong. After all, *every* mathematical object that is indispensable to our best theories of the world is taken to exist. And there is a *huge* amount of objects in this class. Just consider all sorts of mathematical entities that are (allegedly) indispensable to science, from Hilbert spaces and differential equations through complex numbers and topological spaces to a huge variety of geometrical structures. Establishing the *nature* of mathematical objects is simply *irrelevant* for realism. The indispensability of mathematics alone gives the platonist all that he or she needs.

The trouble with this response is that it fails to support realism about mathematics. The *number* of objects the mathematical realist is committed to is not the issue. If all of these objects turned out to be *concrete*, *physical entities*, the nominalist would have nothing to complain about. So, the substantive issue is whether the existing mathematical objects are *abstract*. If the indispensability argument were indeed an argument for realism about mathematics, it would have to establish this latter claim. But the argument doesn't establish that. *At best*, it establishes the *existence* of mathematical entities,[8] leaving aside whether they are abstract or concrete. In fact, given the indeterminacy of reference argument, there's no way of settling either issue (of the existence and of the nature of mathematical objects). After all, as we saw, through an ingenious re-interpretation, the claim that mathematical objects exist comes out true in the modal-structural account of mathematics, even though the account doesn't require the existence of mathematical objects. Thus, we can't say that even the existence of mathematical entities follows from the indispensability argument (basically, the conclusion of the argument can be re-interpreted in a way that the commitment to mathematical objects vanishes). Moreover, mathematical entities can be replaced by *concrete* objects, such as mereological atoms, and supposing that there are enough of the latter, the content of set theory can be recaptured without presupposing sets or (basically) any other abstract entities (see, e.g., Lewis 1991). Thus, whether mathematical entities are concrete or abstract is not decided either.

Furthermore, as we saw, given the indeterminacy of reference argument, it's unclear, for example, what kind of object *numbers* are. Are they sets, Fregean objects, or nodes in a modal structure? We can't determine that, given that all of these descriptions are equally correct. Note, however, that this underdetermination argument can be extended beyond the realm of numbers to include all other types of mathematical objects deemed indispensable in science, such as Hilbert spaces, topologies, continuous functions etc. For these objects can be formulated, for example, in set theory, in category theory or in a modal second-order language, and in each of these cases, the resulting "nature" of the objects would be dramatically different (e.g. sets and categories are not even the same types of things). As a result, the "nature" of mathematical objects (if any) can't be determined simply by pointing out how the objects in question have been defined. Such definitions depend on the *background theory* in which the objects have been formulated, and despite the differences between such background theories, the resulting definitions are equally correct.

In other words, given the indeterminacy of reference argument, the *nature* of mathematical objects is indeterminate: whether such objects are abstract or concrete, and what kinds of objects they are, are issues left open. Hence, on its own, the simple claim that *there are* mathematical objects—the conclusion of the indispensability argument—is *insufficient* to establish realism about mathematics. On its own, that conclusion is compatible with nominalism (if we take, for example, the existing mathematical entities as concrete objects). And if we run a modal-structural interpretation, it's not even clear that that conclusion is warranted anyway. In any case, the indispensability of mathematics is not enough to guarantee realism about mathematics.

5. Undermining the Indispensability Argument

Choosing between indispensability and indeterminacy. Given the tension between the indispensability and the indeterminacy of reference arguments, can we choose between them? The indeterminacy argument seems to be more basic than the indispensability argument, in that it underlines a much more general phenomenon. As presented by Quine, indeterminacy is a feature of our whole language—*including the language we use to formulate our best theories of the world*. Similarly, ontological relativity is also a feature of our conceptual frameworks—*including those used to express our best scientific theories*. So, even when we state that certain mathematical objects are indispensable to science, we are unable to uniquely refer to such objects (in general) and even to determine whether they exist (in absolute terms). As a result, the grounds for the indispensability argument are thoroughly shaken. Hence, if we have to choose between the two arguments, the indispensability argument must go.

But perhaps one could claim that mathematical structuralism saves the day. As we saw, according to the structuralist about mathematics, mathematical objects are only positions in a structure, and such positions have no ontological significance. Whatever objects satisfy the relevant structures will do. So, there's no need to be committed to mathematical objects per se. What counts is the overall *structure*. As a result, our inability to uniquely refer to mathematical objects raises no difficulty for mathematical structuralism, given that the objects are not epistemically significant; *structures* are.

The problem with this move is that there is indeterminacy *even at the level of structure*. What exactly are the structures provided by, say, quantum mechanics? Should we take them to be those given by *group theory* (Weyl), the theory of *Hilbert spaces* (von Neumann) or *q-algebras* (Dirac)? Well, each of these structures is mathematically very different, and they are not at all equivalent. But when used to formulate quantum mechanics, they yield the same empirical results. So, which of them (if any) gives us the structure of the quantum mechanical world (as it were)? There's no *epistemic* way of deciding.[9] Moreover, these three basic structures (group-theoretic, analytic, and algebraic) are formulated in which sort of background theory—set theory, category theory or something else altogether? For reasons discussed above, there is no *absolute answer* here either. But without an answer, it's simply unclear what the *content* of the structuralist's claim about mathematics amounts to. As a result, mathematical structuralism won't solve the problem of reconciling indispensability and referential indeterminacy in mathematics.

Additional problems for the indispensability theorist. Quite independently of concerns regarding indeterminacy, the indispensability argument faces significant problems of its own. And so there are independent reasons why the argument shouldn't be accepted anyway. I'll mention three of them, very briefly.

(a) *Scientific and mathematical practice.* As opposed to the picture suggested by the indispensability argument, scientists *don't accept indiscriminately all the components of a scientific theory*. (That is, scientific practice doesn't seem to support confirmational holism.) For example, to be able to model certain phenomena, scientists often need to introduce idealizations and simplifications—the phenomena in question might be intractable otherwise. As a result, scientists know that the resulting descriptions clearly *don't correspond to the way things are*. Thus, despite the fact that idealizations and simplifications are *indispensable* to our best scientific theories, scientists don't take them to carry ontological commitment (Maddy 1997). Moreover, when one considers mathematical practice, set theorists *don't* look at results in physics to decide whether they should believe in the existence of, say, inaccessible cardinals. This is simply irrelevant to their practice—as it should be (see, again, Maddy 1997). In other words, it looks as though the indispensability argument is actually *incompatible* with scientific and mathematical practice—particularly the confirmational holism that underlies the argument.

(b) *Use of language*. As part of our own language, we recognize that there are terms whose use might be indispensable, but this use offers no reason for us to believe in the existence of the corresponding objects. Our *best theories* of fictional discourse may require us to assert that sentences like "Sherlock Holmes lived in London" are true. But we certainly *deny* any commitment to the existence of Sherlock Holmes! Or consider: "The average mom has 2.3 kids". Does that commit us to the existence of *average moms*? Clearly not! (See Melia 1995.)[10]

(c) *Criteria of existence*. To claim that something exists, it's typically required more than the *indispensability* of the entity in question. The indispensability may be simply due to the need for *expressing* certain claims.[11] To assert the *existence* of something, one requires more than richness in expression. Some sort of *access* to the object in question is required; an access that is *robust*, that can be *refined*, that allows us to *track* the object in question, and to use properties of the object to get to know other properties of the object (Azzouni 1997).[12] Given that mathematical entities do not satisfy such conditions, it's not surprising that, in the end, we find so much controversy as to whether these entities exist. (In fact, it's unclear that we have good reason to believe in their existence in any case!) So, indispensability is just *not* the right way to look at ontological issues.[13]

6. Conclusion

As discussed above, there is a tension between the indispensability argument and the arguments for referential indeterminacy and ontological relativity. Given that the latter arguments are more general and basic—and underlie even the indispensable use of mathematics—they should be preferred. This means giving up on the indispensability argument. But, as noted above, we have good, independent reasons to reject this argument anyway. So, we have good, independent reasons not be platonists either—and so does Quine.[14]

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Resumo

É notório que Quine apresentou o argumento da indispensabilidade para impor a crença de que os objetos matemáticos (tal como as classes) existem, devido a sua indispensabilidade em nossas melhores teorias do mundo (Quine 1960). Ele também defendeu o argumento da indeterminação da referência, de acordo com o qual a referência é indeterminada de uma maneira drástica: dada uma língua, não há uma única relação de referência para ela (cf. Quine 1969a). Neste artigo, defendemos que esses dois argumentos estão em conflito um com o outro. Enquanto que o argumento da indispensabilidade apóia o realismo sobre a matemática, o argumento da indeterminação da referência, quando aplicado à matemática, fornece uma estratégia poderosa em apoio ao anti-realismo sobre a matemática. Concluímos apontando por que o fenômeno da indeterminação da referência poderia ser preferido em detrimento das considerações a respeito da indispensabilidade. No final das contas, mesmo o filósofo quineano não poderia ser um realista (platônico) sobre a matemática.

Palavras-chave
Argumento da indispensabilidade, indeterminação da referência, platonismo, Quine.

Notes

[1] For an insightful and thorough discussion of this argument, see Colyvan 2001.
[2] For elaborations on this style of argument, see e.g. Putnam 1980. Insightful discussions are found in Field 2001, and McCarthy 2002.

[3] Of course, the argument also licenses belief in concrete, observable objects, such as tigers and watermelons, provided they are indispensable to our best theories of the world.

[4] Van Fraassen can also avoid incoherence, by exploring a different strategy, though. He would simply reject the indispensability argument. According to his constructive empiricist view, scientific theories need not be true to be good. The crucial feature is that they are empirically adequate, that is, true with regard to observable phenomena (see van Fraassen 1980). As a result, the fact that mathematical objects are indispensable to our best theories of the world gives no reason to believe in the existence of such objects. After all, for the constructive empiricist, the truth of the theories in question is never asserted—only their empirical adequacy is—and so the empiricist is only committed to the corresponding *observable* objects. Given that mathematical entities are unobservable (again, *pace* Maddy 1990), they are not part of the ontological commitments of the constructive empiricist. The incoherence is dissolved.

[5] The same goes for sets, of course. (Throughout this paper, I'll use 'sets' and 'classes' interchangeably. For our present purposes, nothing hangs on this.)

[6] There are, of course, many additional reconstructions of arithmetic, including those of a nominalist sort, which are not ontologically committed to the existence of numbers. For example, on Hellman's modal-structural interpretation, a statement such as 'There are infinitely many prime numbers' is reinterpreted (translated) in a modal second-order language (roughly) in terms of two other statements. (a) If there *were* structures satisfying Peano arithmetic principles, then it *would* hold in such structures that there are infinitely many prime numbers, and (b) it's *possible* that there are structures satisfying Peano arithmetic principles (see Hellman 1989, for details). Note that statements (a) and (b) do not assert the existence of numbers; if anything, they are only committed to the *possibility* of certain structures. In this way, arithmetic can be developed without presupposing the existence of numbers!

[7] Given that the various reconstructions of arithmetic are *equally correct*, there's no *epistemic* way of choosing between them. There might be *pragmatic* grounds to prefer one reconstruction to another, though (say, based on simplicity, familiarity, explanatory power etc.). But if the choice is purely pragmatic, it gives no grounds to believe in the existence of the corresponding objects. After all, the fact that a theory is simpler or more familiar than another doesn't entail that it is true. Hence, once again, we have no reason to endorse the conclusion of the indispensability argument.

[8] However, even this is controversial; see, e.g., Field 1980, Maddy 1997, the second half of Balaguer 1998, Azzouni 2004, and the next section below.
[9] As noted above, if the choice between such structures is simply *pragmatic*, it will fail to justify any *ontological* commitment to the resulting structures.
[10] Of course, one could always try to paraphrase such sentences without invoking the offending terms. But, alas, such paraphrases are not always available (see Melia 1995).
[11] For a fascinating discussion of this point, see Azzouni 1997, and Azzouni 2004.
[12] According to Azzouni, when these four conditions are met, we have *thick epistemic access* to an object (see Azzouni 1997). Although he doesn't think that such conditions provide criteria for existence—on his view, ontological independence does (see Azzouni 2004)—clearly having thick epistemic access to an object seems to provide some reason for belief in the corresponding object.
[13] For a different way of conceptualizing ontological debates, see Azzouni 2004.
[14] My thanks go to Tony Anderson, Jody Azzouni, Oswaldo Chateaubriand, Mark Colyvan, Newton da Costa, Gary Hatfield, Décio Krause, and Marcos Nascimento for extremely helpful discussions.